

Quantum Moduli Spaces of $N = 1$ String Theories

Tom Banks

*Department of Physics and Astronomy
Rutgers University, Piscataway, NJ 08855-0849*

Michael Dine

*Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064*

Generically, string models with $N = 1$ supersymmetry are not expected to have moduli beyond perturbation theory; stringy non-perturbative effects as well as low energy field-theoretic phenomena such as gluino condensation will lift any flat directions. In this note, we describe models where some subspace of the moduli space survives non-perturbatively. Discrete R symmetries forbid any inherently stringy effects, and dynamical considerations control the field-theoretic effects. The surviving subspace is a space of high symmetry; the system is attracted to this subspace by a potential which we compute. Models of this type may be useful for considerations of duality and raise troubling cosmological questions about string theory. Our considerations also suggest a mechanism for fixing the expectation value of the dilaton.

1. Introduction

Known string models possess flat directions, directions in field space along which the potential vanishes exactly. The subspace of field space on which the classical potential vanishes is called the space of moduli. One of the most important practical problems of string theory is to determine where in the space of moduli the true vacuum state of string theory lies.

In supersymmetric classical vacua, approximate symmetries of the theory guarantee that the superpotential is not renormalized to all orders of perturbation theory. These symmetries are not exact and so we would seem to know very little about possible non-perturbative contributions to the potential; we can not even be sure about their general size in the limit of weak coupling. It is often assumed that effects visible in the low energy theory such as gluino condensation play the dominant role in lifting these flat directions, but in general this is only an assumption. Indeed, it is believed that string theoretic effects may be as large as $e^{-a/g}$, far larger than field theory phenomena.

In [1], it was shown that in many instances, exact discrete symmetries can be used to bound the size of possible stringy non-perturbative effects. In this note, we show that it is often possible to make even stronger statements. In particular, discrete symmetries sometimes permit one to argue that there are *no* stringy non-perturbative effects which correct the superpotential on some subspace of moduli space. In theories with (tree level) anomalous $U(1)$ factors in the gauge group, there can be even stronger constraints. We will present examples of vacua with discrete symmetries and anomalous $U(1)$'s in which one can argue that both stringy and field theoretic corrections to the superpotential vanish. This means that on a subspace of the classical moduli space, there are exact, quantum moduli. We will also describe the dynamics as one approaches this subspace of the moduli space. In others cases we will show that although there is no quantum moduli space, it is possible to describe the dynamics, at weak coupling, completely in terms of well-understood low energy phenomena.

By Stringy Non-Perturbative effects, we have in mind effects which appear in the effective lagrangian at a scale just below the string scale. This lagrangian is highly constrained by symmetries and holomorphy of the superpotential and the gauge coupling function.

In models with anomalous $U(1)$'s, the $U(1)$ is broken by comparatively light fields: the dilaton and some (set of) charged chiral matter fields. Thus there is a range of energies for which it is appropriate to keep these fields in the lagrangian. For these fields, the symmetries are extraordinarily restrictive. In many cases, they forbid any corrections to the superpotential which would lift the classical vacuum degeneracy. Thus any superpotential must be a low energy phenomenon.

In particular, we will be able to realize the suggestion of [2] that there are isolated minima of the potential on moduli space at which SUSY is unbroken and the superpotential vanishes. In [2] it was argued that such points would be dynamically selected in postinflationary cosmology, even if other supersymmetric points with lower vacuum energy exist on moduli space. It was suggested that this cosmological selection principle might resolve the classical vacuum degeneracy and pick out our world from the modular muck of string theory. Unfortunately, our examples show that this is not the case. The vacua that we find do not resemble the real world. In the conclusions we will explore the consequences of this observation, which we feel may be quite profound.

2. Absence of Stringy Non-perturbative Corrections

To see the power of discrete symmetries and holomorphy in restricting the form of stringy non-perturbative corrections to the superpotential, consider the Calabi-Yau manifold defined by the vanishing of a quintic polynomial in CP^4 . There is a subspace of the corresponding moduli space where the model exhibits a large discrete symmetry[3]. This symmetric subspace is described by the quintic $P = \sum Z_i^5$, which respects symmetries under which each coordinate, Z_i , is multiplied by $\alpha = e^{\frac{2\pi i}{5}}$. In addition, there is the symmetry of permutation of each of the Z_i 's. In general these symmetries are R symmetries. Under odd permutations, the superpotential is odd. Under the transformation

$$Z_i \rightarrow \alpha^{n_i} Z_i \tag{2.1}$$

the superpotential transforms as

$$W \rightarrow \prod \alpha^{4n_i} W. \tag{2.2}$$

The classical moduli of these theories are easy to describe. There is a modulus, the Kahler modulus, which describes the overall size of the internal space. This state is invariant under all of the discrete symmetries. Then there are a set of moduli associated with deformations of the complex structure. These are in one to one correspondence with quintic polynomials, and we will denote them by $\mathcal{M}(a_1, a_2, a_3, a_4, a_5)$ where the associated polynomial is $\prod Z^{a_i}$. These fields transform like the corresponding polynomial.

We will consider, first, the case of the $E_8 \times E_8$ heterotic string. In this case, the moduli are paired with 27's of E_6 . At the symmetric point in the moduli space, the discrete R symmetries forbid terms in the superpotential of the form:

$$W = f(S) \quad W = \mathcal{M}f(S). \quad (2.3)$$

Here \mathcal{M} denotes any of the complex structure moduli with vanishing expectation values at this point, and S is the dilaton. So no stringy non-perturbative correction can lift the degeneracy! Indeed, there are numerous other couplings which vanish classically which cannot receive stringy corrections. In particular, all of the massless states of the theory remain massless at this level. This statement applies both to the moduli and to the matter fields in the 27 and $\bar{27}$ of E_6 , as well as the various E_6 singlets.

This is not to say that the vacuum degeneracy cannot be lifted. Effects involving the light fields present in the low energy theory can remove the degeneracy. In the present case, the principle effect is gluino condensation. In [1], we explained how the resulting superpotential is consistent with the symmetries. The point is that light fields can spontaneously break such symmetries. What is striking in this theory is that, at least on this subspace of the moduli space, we can predict the precise form of the superpotential.

These arguments do not significantly restrict the form of corrections to the Kahler potential. Most authors neglect these corrections because they vanish at weak enough coupling. Indeed, at strong coupling, the whole concept of a "light sector" in string theory becomes meaningless, so one might imagine that the Kahler potential in the effective lagrangian for the light sector fields would only be a sensible notion at weak coupling. However, we have argued in [1] that the corrections to the Kahler potential might be large in a region where the field theoretic coupling is weak and the light spectrum is identical to

that at weak coupling. Thus, for our purposes, an exact determination of the superpotential is not yet a complete description of the dynamics on moduli space.

Away from the symmetric point, with one mild assumption, one can still significantly constrain the size of non-perturbative effects. In [1], it was pointed out that if stringy non-perturbative effects can be described by two dimensional field theories (e.g. string instantons), then there is an exact symmetry under which the axion shifts by 2π . This means that stringy non-perturbative effects are necessarily proportional to e^{-nS} , where n is an integer. So, for small coupling, *everywhere in the moduli space*, gluino condensation remains by far the most important effect in lifting the classical vacuum degeneracy. The arguments for this periodicity are admittedly somewhat shaky (though they are probably on a firmer footing than most arguments for S duality, of which this symmetry is a subgroup). As a result, in ref. [1], various discrete gauge symmetries under which S transforms non-linearly were used to constrain the superpotential. In the rest of this paper, however, we will make the stronger assumption of 2π periodicity.

3. An $N = 1$ Model with Non-Perturbative Moduli

If we consider the compactification of the $O(32)$ theory on the same Calabi-Yau manifold, a number of new phenomena arise. First, classically, the theory has no “hidden sector” gauge group. The low energy, $O(26) \times U(1)$ theory is extremely non-asymptotically free. Second, the dilaton transforms under the $U(1)$ symmetry; this leads to an anomaly cancellation of the Green-Schwarz type. As a consequence of this transformation law, there is a Fayet-Iliopoulos D term. This term can be canceled, within perturbation theory, by giving an expectation value to some combination of matter fields. We will see in this section that, beyond perturbation theory this is not necessarily the case. Apart from the assumption of an exact symmetry of 2π shifts of the axion, we will also assume that all terms allowed by symmetries are generated with coefficients $\mathcal{O}(1)$.¹ With these assumptions, one can argue:

1. It is still possible to cancel the D terms. However the moduli space is significantly smaller at the non-perturbative level.

¹ Needless to say, this is a somewhat tricky assumption. Already in field theory, we know cases where it is not quite true.

2. Throughout the moduli space, some of the matter fields (26's) gain mass. In fact, at a generic point, all but one of these fields gain mass. In these regions, one can determine the superpotential for the moduli and dilaton. However, at least on some subspaces, the theory is non-asymptotically free. On these, one can argue that no effect whatsoever lifts the flat directions. Thus the complete non-perturbative string theory still possesses moduli.
3. One can determine how the moduli superpotential and potential behave as one approaches these regions of higher symmetry. In fact, one finds that the moduli are attracted to these regions.

Consider the structure of this model in more detail. At tree level, there are 101 “generations” and one “antigeneration.” A generation consists of a 26 with charge +1, ϕ^+ and a singlet of charge -2 , N^{-2} . The antigeneration contains fields ϕ^- and N^{+2} . The $U(1)$ is anomalous. The anomaly is canceled by assigning the dilaton field, which we write as e^{-S} , charge 200, i.e.

$$e^{-S} \rightarrow e^{200\alpha i} e^{-S}. \quad (3.1)$$

A Fayet-Iliopoulos term is generated in this theory. The sign of this term is such that it can be canceled by one of the fields, N^{-2} .

States charged under $O(26) \times U(1)$ are in one to one correspondence with these moduli. We will denote the 26's as $\phi^+(a_1, \dots, a_5)$ (these are the partners of the complex structure moduli) and ϕ^- . The former, again, transform like the corresponding polynomials while the latter is invariant. The singlets will be denoted as $N^{-2}(a_1, \dots, a_5)$ and N^{+2} . They transform like the corresponding moduli times $(1, 1, 1, 1, 1)(-1)^P$.

As we noted above, in this theory, a Fayet-Iliopoulos term is generated at one loop order with sign such that one of the singlets, N^{-2} , obtains a VEV. In perturbation theory, no superpotential involving the N^{-2} 's alone is permitted. Similarly, there can be no term linear in N^{+2} , ϕ^{-1} or the $O(26) \times U(1)$ singlet fields, E (of which more below) alone, times powers of N^{-2} . As a result, the condition to find a cancellation of the D term, and hence a supersymmetric minimum, is simply

$$g^2 |N_i^{-2}|^2 = \mu^2. \quad (3.2)$$

Note that this statement holds throughout the moduli space.

The description here of the D term cancellation is inherently perturbative. In considering what may happen non-perturbatively, it is important to note that the charge of the field which gets a VEV is opposite to that of e^{-S} . As a consequence, terms of the form $e^{-S}N^r$ can appear non-perturbatively, for some r . Such terms can play a role in lifting flat directions and/or breaking supersymmetry, and will be important in our subsequent discussion.

This fact (that the field which gets a vev has charge opposite in sign to e^{-S}) is quite generic to vacua with anomalous $U(1)$ factors, as long as we stick to the perturbative kinetic term for the dilaton. The anomaly is canceled by writing the dilaton Kahler potential as $-\ln(S + S^* + qV)$, where V is the vector potential superfield and q is the anomaly coefficient. The sign of the charge of e^{-S} is determined by q . The contribution of charged chiral superfields to the D-term is determined by their kinetic term and charge. The overall sign is determined by positivity of the chiral kinetic term. Cancellation of the D-term then fixes the sign of the charged field which gets a VEV to be opposite to that of e^{-S} . However, if as suggested in [1], the real world lies in a region where the dilaton Kahler potential is not given by lowest order perturbation theory, it is possible to reverse this sign. The dilaton contribution to the D term depends on the first derivative of it's Kahler potential, while its kinetic term (whose sign is fixed by positivity) depends on the second derivative. Some implications of this possibility will be discussed in section 5.

Non-perturbatively, there are three important changes in the analysis of the superpotential. First, since e^{-nS} has $U(1)$ charge $200 \times n$, there are now terms which can appear in the superpotential at the high energy scale involving the N^{-2} fields. Second, the Kahler potential may be appreciably modified. This, however, does not qualitatively alter the problem of solving the D and F term conditions at weak coupling. Third, there may in principal be important effects in the low energy theory which can contribute to the superpotential.

At a generic point in the moduli space, there are no discrete symmetries, and a superpotential is permitted which would lift all of the flat directions. For small values of

the moduli, this superpotential has the form

$$W_N = P_{100}(N, \mathcal{M})e^{-S} + \mathcal{O}(e^{-2S}) \quad (3.3)$$

where P_{100} is a polynomial of degree 100 in the N^{-2} fields, and with suitable powers of the moduli to satisfy all of the discrete symmetry constraints. However, for small coupling, we expect field theoretic effects, to be much larger than (3.3); we will see that this is the case.

It is particularly interesting to consider the theory near the symmetric point. In order to have an allowed term in the superpotential, we need to find combinations of N 's and e^{-nS} which transform under the discrete symmetries as $(4, 4, 4, 4, 4)$. Consider first the field $N(1, 1, 1, 1, 1)$. This is a rather symmetric choice. $N^{100}e^{-S}$ is invariant under the gauge symmetry, but does not transform properly under the R symmetries (it is invariant). Similarly, there is no term of the form $N'N^{99}e^{-S}$ which transforms in the correct way. So with respect to the high energy superpotential, $N(1, 1, 1, 1, 1) \neq 0$ is an exact flat direction, perturbatively and non-perturbatively.

However, at scales below $\langle N \rangle$, field theoretic effects can generate a superpotential that lifts this flat direction. Many of the ϕ fields gain mass at this scale – so many that the low energy theory is asymptotically free. Indeed, ϕ 's corresponding to polynomials of the form $(1, 1, 1, 1, 1)$, (one field), $(2, 1, 1, 1, 0)$ (and permutations, for a total of 20 fields), and $(2, 2, 1, 0, 0)$ (an additional 30 fields) gain mass. All together, then, 51 fields gain mass, leaving 50 fields in the low energy theory. An $O(26)$ gauge theory with 50 fields in the fundamental representation, is asymptotically free. One might, then, expect important susy breaking effects could occur, similar to gluino condensation. On the other hand, the low energy is a theory in which, including only renormalizable terms, no superpotential is generated non-perturbatively. We will discuss later the question of whether including non-renormalizable terms, a superpotential is generated, and turn first to a theory in which the low energy theory is not asymptotically free.

There are also perturbative vacua for which the low energy theory is not asymptotically free. These will lead to realizations of the scenario of [2] and to an exact moduli space of $N = 1$ supersymmetric vacua of string theory. Consider, for example, $N(3, 2, 0, 0, 0)$. As before, at the high energy scale, there are no terms which are permitted in the superpo-

tential which can lift the flat direction. To see this, consider the conditions for a coupling to appear in the superpotential:

1. For couplings of the form $N^r e^{-nS}$,

$$(4r, 3r, r, r, r) = (4, 4, 4, 4, 4) \quad (3.4)$$

2. For couplings of the form, $N^r N'(a, b, c, c, c) e^{-nS}$

$$(4r + a + 1, 3r + b + 1, r + c + 1, r + c + 1, r + c + 1) = (4, 4, 4, 4, 4) \quad (3.5)$$

and $r = -1 \pmod{5}$. This has no solutions.

3. For couplings of the form $EN^r e^{-nS}$ or $MN^4 e^{-nS}$ where E are gauge singlet fields and M are the perturbative moduli, there are no solutions, since no single modulus or singlet transforms correctly. (One can obtain the transformation laws of the singlets, for example, from the work of Gepner[4])

The counting of fields which gain mass in this direction is not complicated. Looking at terms of the form $N\phi\phi'$ (terms with higher powers of N and factors of e^{-nS} transform in the same way). Without loss of generality, we can take $\phi(0, 1, a, b, c)$ and $\phi'(0, 0, a', b', c')$. Now it is not difficult to enumerate the possible solutions. One can take $(a, b, c) = (2, 2, 0)$ (and perms), $(a, b, c) = (2, 1, 1)$ (and perms). To each choice, there corresponds a unique ϕ' . All together, then, 24 fields gain mass. This leaves 77 massless 26's in the low energy theory, and the theory at low energies is not asymptotically free.

We have already remarked that away from the symmetric point, there is in general no solution of the D and F term conditions. As a result, there is a potential for the moduli and the fields N . This potential, however, is proportional to e^{-S} , which is small compared to expected effects in the low energy field theory. So let us ignore this and suppose that the D term is canceled by an expectation value for some set of N^{-2} 's. Then, at a generic point in the moduli space all of the ϕ^+ fields can gain mass except ϕ^- , since there are no longer discrete symmetries which prevent the coupling of N^{-2} to any combination of ϕ 's. For small \mathcal{M} , one can think of these terms as arising through terms in the superpotential of the form

$$\phi^+ \phi^+ N^{-2} \mathcal{M}^n. \quad (3.6)$$

Note that the arguments of ref. [5] do not forbid such couplings. Throughout our discussion, we will assume that they are present whenever permitted by symmetries. N^+ also gains mass, through terms of the form

$$N^+ N^+ N^- N^- \mathcal{M}^4. \quad (3.7)$$

Integrating out N^+ , gives a coupling $(\phi^-)^4$ (we will work out the moduli dependence shortly). As a result, the low energy theory of the matter fields possesses no flat directions. A superpotential is generated, and there are minima with unbroken supersymmetry. At the minimum, the superpotential has a non-vanishing expectation value. Since the parameters of this low energy theory depend upon the moduli, this corresponds to the generation of a superpotential for the moduli.

The unique superpotential which respects the non-anomalous R symmetry (which exists when non-renormalizable couplings are ignored) behaves as

$$W_{np} = \frac{(\Lambda)^{71/23}}{(\phi^2)^{1/23}}. \quad (3.8)$$

Here, Λ is the scale of the $O(26)$ theory in which the fermions are massive. It is related to M , the string scale, and the fermion masses, by

$$\Lambda = M e^{-S/b'_o} (\det(m_f/M))^{\frac{1}{b'_o}}. \quad (3.9)$$

Again, the primed quantities refer to the theory with massive fermions; the unprimed quantities to the theory with all fermions massless. In this case,

$$\Lambda = M e^{-S/71} (\det(m_f/M))^{1/71}. \quad (3.10)$$

For weak coupling, this is far larger than e^{-S} . It is natural to ask what happened to the shift symmetry. This was explained in ref. [1]. The point is that the low energy theory has an approximate Z_{142} discrete symmetry, which is spontaneously broken by the ϕ VEV. Shifts in a correspond to changes of this VEV by a discrete phase. Such phenomena can occur in the low energy theory, where even at weak coupling, light fields can gain VEV's and break symmetries. It is also natural to pause and ask what happened to the $U(1)$

gauge symmetry in this expression? After all, one might have been somewhat uneasy about using a broken symmetry to constrain the form of an effective lagrangian. However, this expression does conserve the $U(1)$ once one takes account of the N dependence of Λ . This N dependence can be determined from the renormalization group, and holomorphy:

$$\Lambda = M e^{-\frac{8\pi^2}{b'_o g(M)^2} + \frac{1}{b'_o} \sum \ln(m_i/M^2)} \quad (3.11)$$

where M denotes some large scale such as the Planck mass or string scale, b'_o is the first term in the low energy β function, and m_i are the masses of the fields which gain mass through $\langle N \rangle$. Since $N_i \propto N$, one finds that $\Lambda \propto N^{101/71} e^{-S/71}$, which means that it has $U(1)$ charge $-2/71$. Thus the full superpotential, W_{np} is invariant.

Including the non-renormalizable term, then, the relevant superpotential is

$$W = \frac{(\Lambda)^{71/23}}{(\phi^-)^{2/23}} + \frac{1}{M} (\phi^-)^4. \quad (3.12)$$

A superpotential of the general form

$$W = \frac{\Lambda^a}{\phi^b} + \frac{1}{M} \phi^4 \quad (3.13)$$

has a minimum at

$$\langle W \rangle = (M^{-b} \Lambda^{4a})^{\frac{1}{4+b}}. \quad (3.14)$$

In the present case, this is

$$\langle W \rangle = (M^{-1} \Lambda^{142})^{1/47}. \quad (3.15)$$

The masses, as we will discuss shortly, are determined by the moduli. So is the parameter M . Correspondingly, $\langle W \rangle$ is a function of the moduli – indeed, it *is* the superpotential for the moduli.

To get some idea how this behaves, let's suppose that all of the moduli have comparable VEV's, and ask how the determinant depends on the moduli. Take, again, the case $N = (3, 2, 0, 0, 0)$, and ask how many powers of moduli are required to give mass to each field. The light ϕ fields are of the following types:

$$(3, 2, 0, 0, 0)[2] \quad (3, 1, 1, 0, 0) (3) \quad (3, 0, 2, 0, 0) (3) \quad (3, 0, 1, 1, 0) (3)$$

$$\begin{aligned}
& (2, 3, 0, 0, 0) [2] \quad (2, 2, 1, 0, 0) (3)[2] \quad (2, 1, 2, 0, 0) (3) \quad (2, 1, 1, 1, 0) (3) \\
& (1, 2, 2, 0, 0) (3) \quad (2, 0, 2, 1, 0) (6) \quad (2, 0, 1, 1, 1) \quad (1, 3, 1, 0, 0) (3)[2] \\
& (1, 2, 2, 0, 0) (3)[2] \quad (1, 2, 1, 1, 0) (3)[2] \quad (1, 1, 3, 0, 0) (3) \quad (1, 1, 2, 1, 0) (6) \\
& (1, 1, 1, 1, 1) \quad (1, 0, 3, 1, 0) (6) \quad (1, 0, 2, 1, 1) (3) \quad (1, 0, 2, 20) (3) \\
& (0, 3, 2, 0, 0) (3) \quad (0, 3, 1, 1, 0) (3) \quad (0, 2, 3, 0, 0) (3) \\
& (0, 2, 2, 1, 0) (6) \quad (0, 2, 1, 1, 1). \tag{3.16}
\end{aligned}$$

(77 states in all). Most of these fields can gain mass at first order in the moduli. If higher powers of moduli are required, we have indicated the number in square braces. In each case, this number can be determined by the order of the polynomial required to give the correct transformation properties under the discrete symmetries. E.g. for the first field, one needs a polynomial of degree 15 (after combining $\phi \times N$). This can be provided by one ϕ and two moduli. An example of such a coupling is $\phi(3, 2, 0, 0, 0)\phi(2, 2, 1, 0, 0)N(3, 2, 0, 0, 0)\mathcal{M}(0, 2, 2, 1, 0)\mathcal{M}(0, 0, 0, 2, 3)$. It follows that the determinant of this mass matrix is of order \mathcal{M}^{91} . By the same reasoning, the mass for the N^{+2} field is third order in the moduli. It arises from terms of the form $N^{+2}N^{+2}N^{-2}N^{-2}\mathcal{M}^3$ while the coupling $\phi^-\phi^-N^{+2}$ is fifth order in moduli.

Now to determine the superpotential for the moduli, we can follow [6]. We can view Λ in eqn. (3.11) as a function of the quark masses. This function is analytic, so the expression is valid both for large and small masses. Near the symmetric point, we know the dependence of the masses on the moduli, so in this way we obtain, from eqn. (3.15), the superpotential near the symmetric point. The result of this analysis is that in eqn. (3.13), we have

$$\Lambda \propto \mathcal{M}^{91/71} \quad M^{-1} = \mathcal{M}^7. \tag{3.17}$$

so

$$\langle W \rangle = W(\mathcal{M}) = \mathcal{M}^{189/47}. \tag{3.18}$$

Note, in particular, that $\Lambda \rightarrow 0$ slower than \mathcal{M}^2 , so indeed for small \mathcal{M} the masses are less than Λ .

The structure of this superpotential (and of the resulting potential) is quite interesting. First, it is striking that the system is attracted to the symmetric subspace of the moduli space. Second, the superpotential has a branch cut starting at the origin. This result is consistent with the fact that at the symmetric point, there is a very large number of massless states, and the theory is not asymptotically free. Consequently, the symmetries which we used to show that there were exact flat directions in the high energy effective lagrangian are not spontaneously broken in the low energy theory. The flat directions are exactly flat!

There is a further subtlety which we have ignored up to this point. As $\mathcal{M} \rightarrow 0$, $\phi \sim \mathcal{M}^{-35/47} \rightarrow \infty$. This is consistent with the fact that, to all orders of perturbation theory, there is a flat direction in the $\mathcal{M} \rightarrow 0$ limit. Indeed, in perturbation theory, the discrete symmetries, combined with the $U(1)$ gauge invariance, forbid any operator of the form $\phi^{-n} N^{-2m}$ or $\phi^{-n} \phi' N^{-2m}$, etc. However, non-perturbatively, there are operators which can appear and lift the flat direction. For example

$$(\phi^-)^3 \phi(3, 2, 0, 0, 0) N(3, 2, 0, 0, 0)^{99} e^{-S} + \text{perms} \quad (3.19)$$

is consistent with all of the symmetries. It involves no factors of the moduli.

Once the moduli are sufficiently small, this term will become important. However, its effects still tend to zero rapidly as the moduli tend to zero. For non-zero \mathcal{M} , the field $\phi(3, 2, 0, 0, 0)$ has a mass of order \mathcal{M}^2 , but the coupling $\phi(3, 2, 0, 0, 0)^2$ is of order \mathcal{M}^3 . Integrating out $\phi(3, 2, 0, 0, 0)$, then, leads to a sixth order term in the ϕ superpotential of the form:

$$W_6 = \frac{1}{\mathcal{M}} (\phi^-)^6 e^{-2S}. \quad (3.20)$$

In the limit of very small \mathcal{M} , neglecting the ϕ^4 term, we can take the superpotential to be

$$W = \frac{\mathcal{M}^{91/23}}{\phi^{2/23}} + \mathcal{M}^{-1} \phi^6 e^{-2S}. \quad (3.21)$$

Solving for the minimum,

$$\phi \sim \mathcal{M}^{114/140} \quad W \sim \mathcal{M}^{12626/3220}. \quad (3.22)$$

So the potential still goes rapidly to zero, but not quite as rapidly as before.

Finally, it is worth noting that absence of asymptotic freedom is not an essential requirement for the vanishing of the potential. In the classical flat direction with $N(1, 1, 1, 1, 1) \neq 0$, a repetition of the analyses above gives a non-perturbative superpotential behaving as

$$W_{np} \propto \mathcal{M}^{108/47}$$

Thus, in this case as well, the potential vanishes rather rapidly for small \mathcal{M} , and the symmetric point is the minimum.

3.1. Discussion

We have thus exhibited what appears to be a quantum moduli space of string theory. Symmetries constrain the quantum mechanically exact superpotential in the effective lagrangian below the string scale to vanish on a subspace of moduli space. Low energy field theory effects, which might spontaneously break these symmetries are absent because the low energy theory is infrared free.

There is a possible loophole in this argument, which has been revealed by recent work on $N = 2$ supersymmetric string and gauge theories[7]. In such theories one occasionally find points in moduli space at which soliton states become massless, while their classical size remains smaller than their Compton wavelength. If the solitons are magnetically charged under a perturbatively infrared free gauge theory, they will change the sign of the β function for weak coupling. The infrared dynamics will be driven to a strong coupling fixed point, which is as yet poorly understood. Can we rule out the occurrence of such phenomena on our putative quantum moduli space?

In analyzing this question we will make the assumption that, for four dimensional compactifications, such effects manifest themselves as singularities in the coefficients of string perturbation theory. Indeed, a massless magnetic soliton state should show up in the vacuum polarization function of the gauge fields. For the symmetric point in complex structure moduli space, both at large radius, and at the Gepner point, there are no such singularities. Local holomorphy of gauge kinetic functions then shows us that there are no such singularities in a finite radius ball (in the space of all moduli including the string

coupling) around the zero coupling symmetric point (at *e.g.* the Gepner radius). Within this ball our previous analysis is valid, and there are no contributions to the superpotential on our quantum moduli space. Since the superpotential is locally holomorphic, it vanishes everywhere on the quantum moduli space. Note that this does not imply that there are no massless soliton points in moduli space. We learn only that these points are not on the quantum moduli space for sufficiently weak coupling, and that they do not effect the superpotential on the part of moduli space we have explored. In other words, our results about the vanishing superpotential can be analytically continued through any massless soliton points. The soliton mass will be a holomorphic function of the moduli which might happen to have a zero on a subspace of moduli space with finite complex codimension. Probably, if such points exist, they define new branches of the moduli space that connect onto the branch we have studied along this submanifold of zeroes.

4. Directions in Which the System is Repelled From the Symmetric Point

So far, we have studied in some detail an example in which a subspace of the classical moduli space survives in the full quantum theory. This subspace is a space where the theory exhibits a high degree of symmetry, and it is perhaps not surprising that, at least for weak string coupling, we have been able to show that it is a domain of attraction in the moduli space. In this section we consider an example where the opposite is true. At weak coupling, the degeneracy is lifted by non-perturbative effects. However, the system is repelled from the region of high symmetry.

The model is the four generation version of the quintic in CP^4 discussed in ref. [3]. This model is obtained by modding out the Calabi-Yau space we have discussed earlier by two freely acting Z_5 symmetries. The resulting model has four “generations” (ϕ^+ and N^{-2}) and one “antigeneration” (ϕ^- and N^{+2}). Following ref. [3], we denote the generations by $\psi_o, \psi_2, \psi_{-2}, \psi_1$ and ψ_{-1} (where ψ can be either ϕ^+ or N^{-2}); we will denote the antigeneration by $\bar{\psi}$. The model possesses two discrete symmetries, referred to as W and T . Under W , $\psi_n \rightarrow \psi_{-n}$, while $\bar{\psi}$ is neutral. Under T , $\psi_n \rightarrow e^{2\pi i n/5} \psi_n$. W is an R symmetry under which the superpotential is odd. T is an ordinary symmetry.

At weak coupling, we can analyze this model in much the same way as we analyzed the models above. Again, in this theory, a D term is generated at one loop, with a sign such that it can be canceled by an expectation value for N^+ . The field e^{-S} now has charge 4 under the $U(1)$. At the classical level, there is no obstacle to giving expectation values to the N fields so as to cancel the D term. Non-perturbatively, the symmetries permit additional terms in the high scale effective lagrangian which would lift the degeneracy, such as

$$(N_2 N_2 e^{-S})^5 + (2 \rightarrow -2) + (N_1 N_1 e^{-S})^5. \quad (4.1)$$

However, no terms are permitted at this level which lift the N_o direction.

As we will shortly see, however, field theoretic effects are far more important than the highly suppressed stringy effects considered above. Let us examine the structure of the classical superpotential. As explained in [3], the non-vanishing cubic terms are

$$\psi_o^3, \psi_o \psi_2 \psi_{-2}, \psi_2 \psi_{-1} \psi_{-1}, \psi_{-2} \psi_1 \psi_1, \psi_2 \psi_2 \psi_1, \psi_{-2} \psi_{-2} \psi_{-1}. \quad (4.2)$$

Now consider a particular classical direction, $\langle N_2 \rangle \neq 0$. In this direction, all of the ϕ_n 's gain mass at tree level. ϕ^- , however, remains massless. So at low energies, the theory has the structure of an $O(26)$ SUSY gauge theory with a single 26. At a generic point in moduli space, just as in our earlier study, there are no flat directions. In this case, however, the coefficient $\frac{1}{M}$ in the superpotential, $W = \frac{1}{M}(\phi^-)^4$ is linear in the moduli. Repeating the analysis of the preceding section, a non-perturbative superpotential is seen to be generated. It has the same form as eqn. (3.12). However, in this case, Λ is independent of the moduli. Again, the minimum is given by eqn. (3.15). This corresponds to a superpotential for the moduli which behaves as $\mathcal{M}^{1/47}$. The resulting potential blows up as one approaches the symmetric point, almost as fast as $\frac{1}{|\mathcal{M}|^2}$.

This behavior is particularly easy to understand in the present case. At the symmetric point, no additional fields transforming under $O(26)$ becomes massless; the non-perturbative superpotential has the same structure as at the generic point. Thus for the effective theory at (or near) this point, the effective superpotential for N^{+2} , ϕ^- and \mathcal{M} has the structure

$$W_{eff} = \frac{\Lambda^{71/23}}{(\phi^-)^{2/23}} + (\phi^-)^2 N^{+2} N^{+2} \mathcal{M}. \quad (4.3)$$

It is easy to see that this superpotential has no supersymmetric minimum. It is possible to lower the energy by letting \mathcal{M} become large; this is precisely the repulsion we found above. However, for sufficiently large \mathcal{M} , our analysis breaks down. Thus, it is likely that there are supersymmetric minima of the potential on moduli space at a finite distance (in string units) from the symmetric point. Generically, the potential at such points will be negative [2], and the moduli will not come to rest at these minima of the potential after a period of inflation. Thus, it might be that this entire region of moduli space is ruled out by cosmological considerations. We cannot of course rule out the possibility that far from the symmetric point there is a nonsupersymmetric minimum with nonnegative cosmological constant, which would be an attractor for the dynamics of postinflationary cosmology.

Other directions (VEV's for N) can be studied in a similar way. The analysis is, in some cases, more complicated because there are more light fields, but the results are similar.

5. Some Strange Possibilities

So far, the spirit of our discussion has been to work at very weak coupling and to establish the existence of moduli by working perturbatively in some of the (other) moduli. In this way, we have now established the existence of exact quantum moduli for this theory, corresponding at weak coupling to the original dilaton and to the radial dilaton.

It is natural to ask what sorts of phenomena might occur as we move in towards stronger coupling. One interesting possibility is the following. The kinetic term for the dilaton has the structure

$$\int d^4\theta K(S + S^\dagger + V). \quad (5.1)$$

The D term is just $\frac{\partial K}{\partial S}$. A priori, we know of no reason why this might not vanish for some value of S , call it S_o .² At this point, the potential has the structure:

$$V = (m_V s + \sum q_i |\phi_i|^2)^2. \quad (5.2)$$

² Since the Kahler potential is negative, convex and increasing as $S \rightarrow \infty$, its derivative could only vanish at finite S_0 if its second derivative were to change sign. This would imply a negative kinetic energy for the dilaton were it not for the possibility of kinetic mixing between the dilaton

Here $m_V^2 = \frac{\partial^2 K}{\partial S^2}$ is the mass of the vector meson, and $s = \text{Re}(S - S_o)$ (up to a rescaling to give a canonical kinetic term). There are several interesting features of this result:

1. At $s = 0$, the theory is supersymmetric. The dilaton is completely absorbed by the supersymmetric Higgs mechanism.
2. For $s > 0$, fields with one sign of the $U(1)$ charge get expectation values; for $s < 0$, fields with the other sign do.

There are a number of possibilities for the physics at such a point, depending on the particle content of the model. Here we list some of them (we do not claim to know string vacua which realize every one of these possibilities, but we know of no general argument that such vacua cannot exist).

1. There are no flat directions classically besides the dilaton. In this case, at one loop, the dilaton is fixed at S_o . Supersymmetry may be unbroken in the low energy theory or it might be broken; if it is broken, the scale of the breaking is of order e^{-S_o} . As we have argued elsewhere, it is perfectly possible that, even though perturbation theory is not good for the Kahler potential, this number is small.
2. There may be several fields with one sign of the charge, such that, classically, the D term is canceled. Non-perturbatively, terms of the form $e^{-S} N^r$ will lift these flat directions. Thus, at weak (but non-zero) coupling, there will be no ground state. However, at strong coupling, the dilaton and/or fields with the opposite sign of the $U(1)$ charge (for which no non-perturbative superpotential may be permitted since e^{-S} has the wrong sign of the charge) may have expectation values such that the D term vanishes. In this case we would have an analytic moduli space for $\text{Re } S < S_o$ but no vacuum state for larger values of the real part of S . This would provide a counterexample to the generic analyticity of supersymmetric moduli spaces.

Clearly one can go on to enumerate further possibilities, and one can speculate on connections to recent developments with duality symmetries. We will leave this for future work, and just note that the vanishing of the D term could well play an important role in determining the fate of the dilaton.

and other moduli. We assume that such mixing occurs in the following speculations. We thank V. Kaplunovsky for a discussion of this point.

6. Conclusions

In this paper we have found examples of supersymmetric stringy vacuum states with vanishing cosmological constant. In [2], such states were suggested as the natural postinflationary ground states of string theory for the nondilatonic moduli. There it was emphasized that the modular potential in such states would leave the dilaton direction flat. It was suggested that lower energy nonperturbative gauge dynamics could lift this degeneracy. If this led to a stable vacuum with vanishing cosmological constant, it would have to break SUSY.

The states that we have discovered do not have these properties. They have no SUSY breaking low energy dynamics. Indeed we have argued that they have an exact quantum moduli space of supersymmetric vacua with vanishing cosmological constant. Apart from the disappointment of finding that the “cosmological vacuum selection principle” of [2] does not uniquely select the real world from among the myriad points in moduli space, the existence of these states poses a problem of principle for string theory. Why is the world we see around us not in one of these highly stable states³?

Superstring theory has long been known to have a quantum moduli space of vacua which do not resemble the real world. These are states with extended spacetime SUSY. It has seemed possible to imagine that these states are topologically disconnected from the part of moduli space in which our world lies, and that some sort of nonperturbative anomaly might afflict one and not the other. Alternatively, one might attempt to rule these states out cosmologically, since the quantum effective potential vanishes identically on the moduli space of extended SUSY ground states. Thus, if they are truly disconnected from the $N = 1$ classical moduli space, they could never be the result of a period of inflationary expansion.

By contrast, the quantum moduli space that we have discovered lies right in the middle of the parts of classical moduli space where potentials are generated. They do not look

³ It is not even clear that there is an anthropic answer to this question. Such an argument would depend on the postinflationary history of a universe which asymptotes to one of these states and would quickly become enmired in unanswerable questions about whether there can be life forms radically different from ourselves.

terribly different than the classical vacua which resemble the world we live in. The only kind of cosmological argument that might favor one over the other is the observation of [8], that superstring inflation requires mild fine tuning. In order to have ~ 100 e-foldings of inflation, the curvature at the maximum of the potential has to be one or two orders of magnitude smaller than one would have guessed on the basis of dimensional analysis. Thus there may be few places on moduli space where inflation occurs, and a world like our own (rather than some point in the supersymmetric quantum moduli space) might be selected by the accident that it lies near one of these inflationary maxima. *A priori* it seems no less likely for one of the exact supersymmetric quantum ground states to lie near an inflationary maximum of the potential than does our own world. Thus, until we understand a lot more about the potential on moduli space than we do at present, this will not be a terribly satisfactory explanation of the properties of our world.

The work of Susskind[9] suggests an alternate way to understand the instability of vacua with extended SUSY. Susskind[10] has speculated that black hole evaporation can lead to a unitary S-matrix only in four dimensions. If this is the case, and if string theory is consistent, then in higher dimensional ground states, the information loss paradox of black holes can only be resolved by the existence of remnants. But in [9] Susskind argues that in a theory with remnants flat space is unstable to decay into a gas of remnants (perhaps with an enormously long lifetime). Thus, higher dimensional vacua might be unstable. Vacuum states with extended SUSY are continuously connected (even quantum mechanically) onto flat higher dimensional vacua, and would thus be unstable as well.

While we are not claiming that this particular line of reasoning is the resolution of the problem of stable supersymmetric vacua, we are intrigued by the possibility that it raises. The resolution of the puzzles of black hole evaporation is a problem in nonperturbative string theory, and may well involve intrinsically stringy dynamics which is not captured by low energy field theory[11]. The preceding paragraph suggests that the stability of string ground states may also be a problem that cannot be analyzed completely by low energy effective field theory.

If this is the case, then our proof of nonperturbative stability of a family of $N = 1$ string vacua is incomplete, and might well be incorrect. The problem of resolving the

degeneracy of classical string ground states would not reduce to the determination of the quantum effective potential on the classical moduli space, but would have to be addressed with the full nonperturbative apparatus of string theory (presently unknown). Our discovery of exact (at the level of analysis of effective field theory), four dimensional, $N = 1$ supersymmetric vacua of string theory is a disquieting indication that the construction and nonperturbative solution of quantum string theory may be a prerequisite to answering even the most basic questions about the phenomenology of the theory.

Another (perhaps academic) question that is raised by the existence of a quantum moduli space of $N = 1$ vacua is whether the methods of [12] and particularly [13] could be generalized to give an exact computation of the gauge kinetic functions on the moduli space. For general $N = 1$ compactifications, there does not seem to be any sensible extension of [12][13]. Superpotentials will be generated in all directions in classical moduli space, and for coupling $(\frac{g^2}{4\pi})$ of order one, there will be no sensible separation between the moduli and the massive fields of the theory. For such compactifications, the notion of moduli is an approximate one, and exact analytic computations would seem impossible⁴. Another indication of this is that we generically expect $N = 1$ compactifications to contain strongly coupled gauge sectors which will generate terms in the effective action which are not periodic in the model independent axion. The method of [12] and [13] was to map one string theory on another in such a way that the coupling constant of the original theory became a geometrical modulus of the dual theory. Quantum calculations were then reduced to tree level calculations. But tree level string calculations never “spontaneously break” the periodicity of geometrical moduli in the way that strong coupling effects such as gaugino condensation break the axion periodicity.⁵

On the other hand, no such arguments prevent us from generalizing [12][13] to the quantum moduli spaces of $N = 1$ compactifications that we have discovered. It seems

⁴ We thank N. Seiberg for discussions about this point.

⁵ The only apparent loophole in this argument that duality cannot give us an exact computation of gaugino condensation is the possibility that the duality mapping appropriate for $N = 1$ maps the dilaton onto a branched cover of the geometrical moduli space of tree level string theory. Possibly this loophole can be closed by considering how the map must vary as a function of those moduli which break the low energy gauge group to an abelian subgroup.

likely to us that these methods might lead to exact calculations of gauge kinetic functions on subspaces of these moduli spaces.

Finally, the models we have considered here provide a natural arena in which to discuss questions of duality in $N = 1$ theories. Generically, we expect that the classical flat directions of $N = 1$ theories are lifted, and the discussion of duality might become somewhat murky. Models with exact non-perturbative moduli are a natural arena in which to explore such dualities. Indeed, examples of $N = 1$ dualities which have been uncovered recently[14][15] seem to possess this feature. On the other hand, it may not be so easy to guess the duals of some of the models we have considered here. It is natural to speculate, for example, that under S duality, the low energy gauge theory limit of a particular string vacuum should go over to its Seiberg dual [16]. In this paper, we considered at some length a model with gauge group $O(26)$ and $N_f = 77$. According to [16], the dual theory has gauge group $O(55)$. It is not easy to see how such a gauge group can arise in any string theory.

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